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# ВЛИЯНИЕ СТЫКОВ НА ПОВЕДЕНИЕ СЕГМЕНТНОЙ ОБДЕЛКИ ТОННЕЛЯ КВАЗИ-ПРЯМОУГОЛЬНОГО ОЧЕРТАНИЯ

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***Аннотация:*** Рост численности населения в крупных городах по всему миру требует существенного улучшения транспортной инфраструктуры, которое может быть достигнуто за счет развития подземного пространства городов. Речь идет о расширении строительства дополнительных линий метрополитена и увеличении пропускной способности тоннелей. В области подземного строительства наметилась тенденция к использованию круглых и прямоугольных форм поперечного сечения тоннелей, последние сочетают в себе преимущества обеих форм. Квази-прямоугольное поперечное сечение является частным случаем таких сечений. Изучению формирования напряженного состояния таких обделок посвящены лишь отдельные научные работы. Представлена методика расчета напряженного состояния сборной обделки квази-прямоугольного поперечного сечения с учетом наличия связей между отдельными сегментами, характеризующимися конечной жесткостью. Исследование направленно на изучение влияния вращательной жесткости соединения двух смежных сегментов, а также их количества на напряженное состояние тоннельной обделки. Полученные результаты показывают, что по мере уменьшения жесткости вращения и увеличения числа сегментов обделки внутренние силы в обделке тоннеля уменьшаются, в то время как радиальные смещения увеличиваются, то есть наблюдается снижение жесткости обделки. При дальнейшем увеличении количества сегментов обделки наблюдается снижения влияния их количества на интегральные показатели напряженного состояния обделки и ее радиальные смещения.

***Ключевые слова:*** квази-прямоугольная форма, тоннель, расчетный метод, сборная обделка, жесткость, напряжения, напряженное состояние, отпор массива.

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# Effect of joints on the behavior of a segmental sub-rectangular tunnel lining

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***Abstract:*** The increasing population in major cities around the world requires a significant improvement in transport infrastructure, which can be achieved by developing the underground space of cities. This involves expanding the construction of additional underground lines and increasing the capacity of tunnels. In the field of underground construction, there is a trend towards cross-sectional shapes that combine circular and rectangular tunnels. This innovative shape combines the advantages of both shapes. The sub-rectangular cross-section is a special case of such cross-sections. Only separate scientific works are devoted to the study of stress state formation of such linings. The paper presents a method of calculating the stress state of a prefabricated lining of a sub-rectangular cross-section, taking into account the presence of connections between individual segments characterised by finite stiffness. The study is aimed at investigating the influence of the rotational stiffness of the connection of two adjacent segments, as well as their number on the stress state of the tunnel lining. The results obtained in this paper show that as the rotational stiffness decreases and the number of lining segments increases, the internal forces in the tunnel lining decrease, while the radial displacements increase, i.e. the stiffness of the lining decreases. With further increase in the number of lining segments, a decrease in the influence of their number on the integral indices of the stress state of the lining and its radial displacements is observed.

***Key words:*** sub-rectangular tunnel, joint stiffness, joint location, number of joints, stiffness, stresses: stress state, lining behavior.

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## Introduction

The population growth in major cities around the world demands significant development in transportation infrastructure, while the available surface areas in urban continue to shrink. Hence, the development of metro lines is essential in large cities. Twin circular tunnels with one-way traffic in each single tunnel are usually the typical metro line solution. Compared to a large single circular tunnel, twin tunnels have technical advantages such as less surface settlement and smaller internal forces induced in the tunnel lining [1]. However, the twin tunnels solution causes a greater surface settlement trough. The sub-rectangular tunnel, which was studied by Huang et al., 2018 [2], is a combination of the circular and rectangular tunnels. Compared with a large circular tunnel, a sub-rectangular tunnel allows increasing the efficiency of underground space utilization by around 20%, and the tunnel depth can also be reduced by a corresponding proportion [1].

Studies on the behavior of sub-rectangular tunnel linings can be conducted using experimental methods [3], analytical methods [4–6], and numerical methods [7, 8]. It is worth noting that a large amount of research is devoted to the lining structures in the mining industry [9, 10]. The practice of assessing the stress–strain behavior of the soil mass and lining of the St. Petersburg underground has succeeded, which is noted in the researches [11–13].

Huang et al., 2018 [2] conducted a study on the behavior of sub-rectangular tunnel linings (Fig. 1) through a full-scale test considering the self-weight of the structure. A new loading configuration was developed to experimentally load the full-scale sub-rectangular tunnel lining, enabling the comprehensive assessment of the mechanical properties of the segmental tunnel lining under full loading conditions for the first time. The experimental results were compared with numerical simulations using the Abaqus program. The results indicated that the maximum positive moment was observed near the crown and the bottom part of the tunnel lining, while the maximum negative moment was observed at the shoulder parts.

Zhu et al., 2018 [14] conducted numerical simulations to study the mechanical behavior of sub-rectangular tunnel linings. In the study, the behavior of circular, subrectangular, and rectangular tunnel lining with the same external dimensions were clarified. The results indicated that circular tunnels exhibited high load-bearing capabilities but were less economically efficient compared to sub-rectangular tunnels. Meanwhile, rectangular tunnels pro-

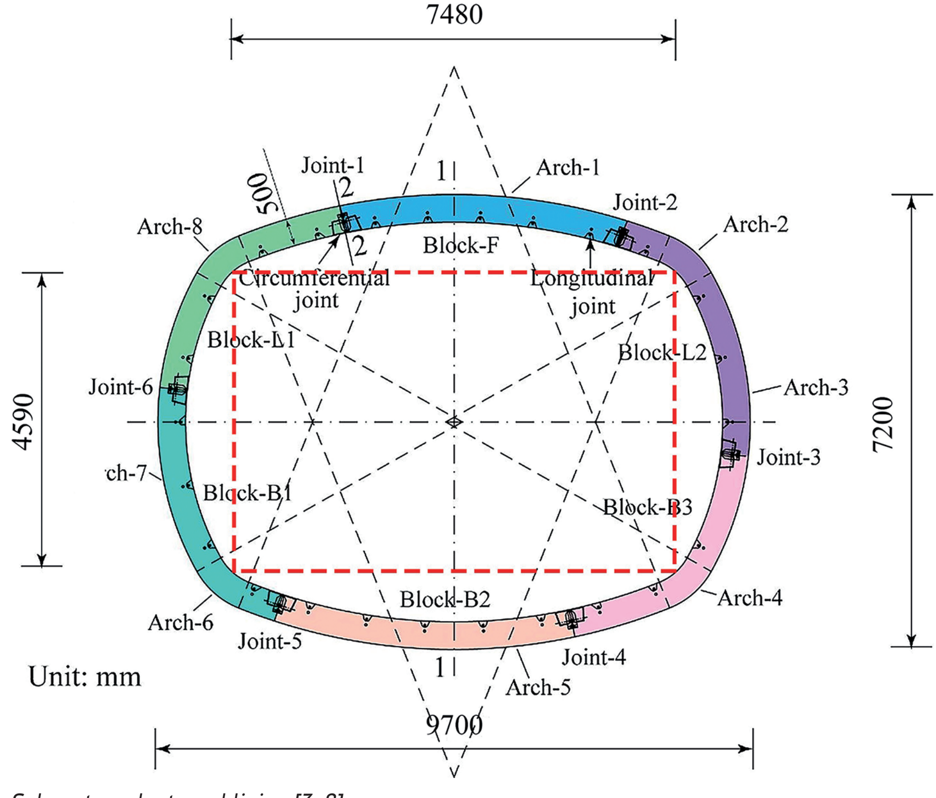


*Fig. 1. Sub-rectangular tunnel lining [3, 8]*
*Рис. 1. Квази-прямоугольная обделка тоннеля [3, 8]*

ved to be unfavorable for load transfer and required additional reinforcement at the corners.

Zhang et al., 2020 [4] investigated the influence of rotational stiffness on the behavior of joints in the segmental lining of a quasi-rectangular tunnel with an interior column. An equation expressing the relationship between the joint rotation angle, bending moment and axial force is proposed for a specific tunnel geometry, based on numerical model results and experimental data. Zhang also concludes that the width of the interior column and the shear stiffness of the joint significantly affect the lining behavior.

Zhang et al., 2021 [15] conducted studies to optimize longitudinal joints (enhancing concrete properties, increasing the number and diameter of bolts, and expanding the lever arm distance between bolts and the compressed zone) in a quasi-rectangular shield tunnel with an interior column. Results from experiments and numerical simulations performed by Zhang et al, 2021 [15] demonstrate that increasing the distance between bolts and the compressed zone most effectively improves the performance of the joints.

Although many authors have previously studied the influence of joint stiffness, joint position, and the number of joints on the performance of structures. These studies have been conducted on the tunnel lining with circular cross-sections. In the study of Zhang et.al, the impact of joint stiffness on the behavior of the quasi-rectangular tunnel lining with an interior column was investigated. However, the influence of the joint position and the number of joints was not addressed in Zhang's study. On the other hand, in the quasi-rectangular cross-section tunnel studied by Zhang, which includes an interior column, the presence of this column leads to differences in the mechanical behavior compared to a sub-rectangular tunnel lining without an interior column. Therefore, the effect of joints on the behavior of sub-rectangular tunnel linings (without column) is investigated using numerical methods in this study. This approach allows a more detailed analysis of the structural behavior under variations in the rotational stiffness of joints, the number of joints and the position of joints.

**The stiffness of segmental lining**

Currently, there are four practical calculation methods utilized in the design of segmental tunnel linings: (a) the routine method, which completely disregards the impact of segmental joints; (b) the multi-hinge ring method, where joints are treated as free hinges [16]; (c) the modified routine method (MRM), which assumes a uniformly rigid ring with an average rigidity derived from a rigidity reduction factor and bending moment transmission [17, 18]; and (d) the beam-spring model (BSM), which treats segments as beams and joints as springs [19—21]. Generally, the existence of joints results in a decrease in the stiffness of the segmental lining. This suggests that the segmental lining's deformability is higher compared to that of a continuous lining [5, 22], this fact is also reflected in the works [23—25].

In the structural analyses, a longitudinal joint can be conceptualized as a combination of an axial spring (KA), a rotational spring (KRO), and a radial spring (KR), as shown in Fig. 2.

According to the research conducted by Do, 2014 [19], under axial stiffness conditions ranging from 10 to 3000 MN/m, it is revealed that the axial stiffness of the joint has an insignificant impact on the behavior of the segmental tunnel lining. Additionally, in this study, when evaluating the influence of the radial stiffness of joints with and without padding (made of rubber, bitumen material) with varying stiffness. Do, 2014 [19] also points out that

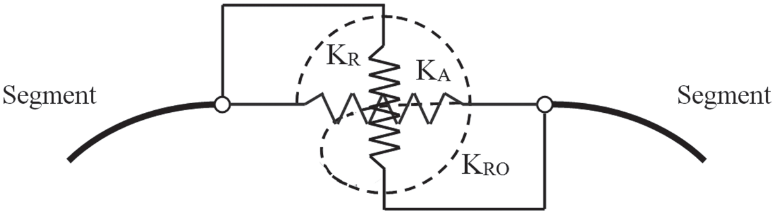


*Fig. 2. Axial stiffness ($K_A$), radial stiffness ($K_R$) and rotational stiffness ($K_{RO}$) of a longitudinal joint [22]*

*Рис. 2. Осевая жесткость ($K_A$), радиальная жесткость ($K_R$) и жесткость при вращении ($K_{RO}$) продольного стыка $K_{RO}$ [22]*

radial stiffness does not significantly affect the behavior of the segmental tunnel lining.

Therefore, this study focuses on evaluating the influence of the rotational stiffness of longitudinal joints on the behavior of the joints and internal forces induced in segmental tunnel lining.

The rotational stiffness value is the bending moment per unit length needed to induce a unit rotation angle along the joints of the segmental lining. It can be simply estimated using the following equation [14]:

$$K_{RO} = \frac{M}{\varphi}. \tag{1}$$

Here, $M$ is the bending moment per unit length of joint; kN.m/m; $\varphi$ is the unit rotation angle of joint, degree.

*Determination of rotational stiffness according to Leonhardt*

Stiffness of the joint is determined by the formula:

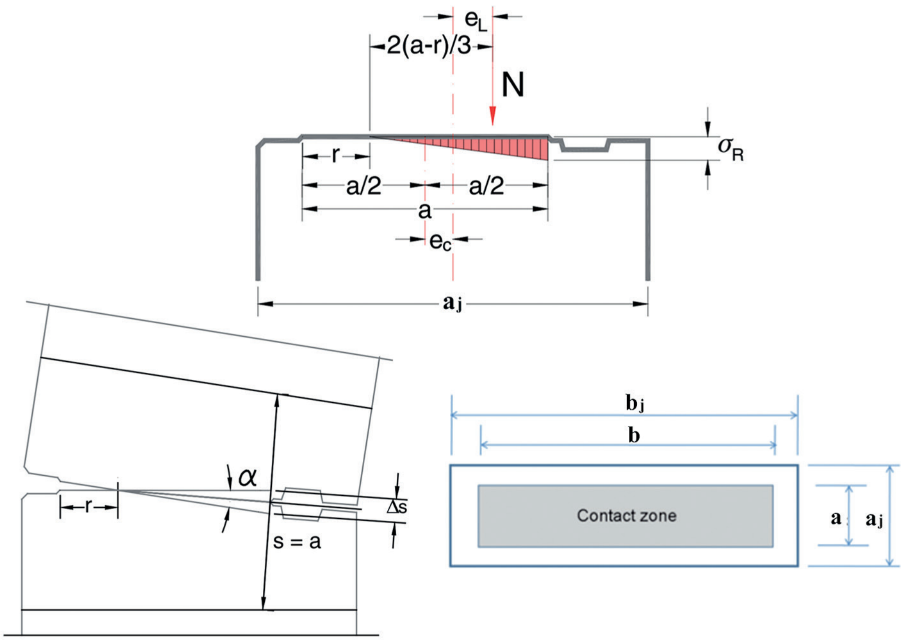


*Fig. 3. Approaches for estimating the torsional resistance of concrete hinges $e_L$ = M/N – eccentricity of the normal force; $e_c$ – eccentricity of the hinge neck*

*Рис. 3. Подходы к оценке сопротивления скручиванию бетонных петель $e_L$ = M/N – эксцентриситет нормальной силы; $e_c$ – эксцентриситет относительно центра шарнира*

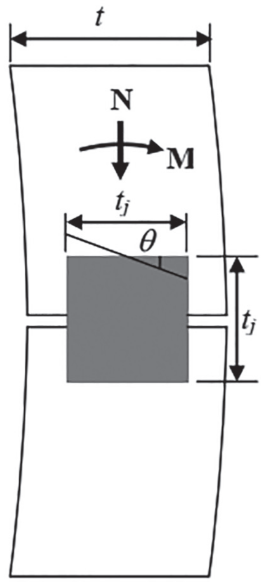


*Fig. 4. Cross-section of the longitudinal joint [23]*
*Рис. 4. Поперечное сечение продольного стыка [23]*

$$k_\varphi = 1.125a^2bE_cm(1-2m)^2. \quad (2)$$

Here, a is the width of the contact surface in the joint, m; *b* is the length of the contact surface in the joint, m; $E_c$ is Young's modulus of the lining material, kN/m$^2$; *m* is the eccentricity ratio, *m = M/aN*; *M* and *N* are the bending moment and normal force in the joint, respectively; $\alpha$ is the torsional angle.

*Determination of rotational stiffness according to Janssen*

The rotational stiffness of the segmental joints is determined using Janssen's formulas. Janssen's joint posits that the contact area can be modeled as a concrete beam. This beam depth equals the width of the joint contact area, and its height matches the contact height of the joint (Fig. 4). The joint opening is considered as the concrete beam is unable to bear any tensile stresses.

For an increasing rotation of the joint, two stages can be identified [14, 26]:

1. Closed joint (rotation $\theta \le \dfrac{2N}{E_cbt_j}$);

2. Opened joint (rotation $\theta > \dfrac{2N}{E_cbt_j}$),

where *b* is the length of the surface part in the joint, m; $E_c$ is Young's modulus of the lining material, kN/m$^2$; $t_j$ is the height and width of the contact surface in the joint, m; *N* is the normal force, kN/m; $\theta$ is the rotational angle of joint, degree.

Rotational stiffness throughout the operational phases of the joint:

$$K_{RO} = \frac{b \cdot t_j^2 \cdot E_c}{12}, \text{ Closed joint.} \quad (3)$$

$$K_{RO} = \frac{9bt_jE_cM\left(\dfrac{2M}{Nt_j}-1\right)^2}{8N}, \text{ Opened joint.} \quad (4)$$

*Rotational stiffness of the joint in a sub-rectangular lining.*

Zhang, 2020 [4] presented a third-degree formula for bending moments and axial forces with corresponding polynomial coefficients (5). This formula is developed to determine the rotational angle of the joint in the segmental lining of a quasi-rectangular tunnel with an interior column. These results are built upon the convergence of numerical methods and experimental outcomes.

$$\varphi(M,N) = M^3 + M^2 \cdot N + $$
$$+ M \cdot N^2 + M^2 + M \cdot N + M. \quad (5)$$

To avoid excessive length in the article, the values of polynomial coefficients are not presented; interested readers can refer to Zhang's article for details [4]. Zhang [4] also notes that the formula is associated with a specific tunnel geometry. However, Zhang et al. [4] are confident that parameters related to the tunnel's shape could be integrated into the formula in future developments. The rotational stiffness value can be determined according to the formula (6), as follows [16]:

$$K_{RO} = \frac{M}{\varphi(M,N)} = \frac{1}{M^2 + M \cdot N + N^2 + M + N} \quad (6)$$

It can be observed that, in reality, the rotational stiffness of joints within a segmental ring varies and depends on the magnitude of moments and axial forces within the joint. However, for the purpose of simplification, the rotational stiffness of the joints is assumed to be uniform across all joints in this study.

To depict the correlation between structural forces and displacements of the lining based on joint rotational stiffness, the dimensionless factor termed the rotational stiffness ratio, denoted as $\lambda = (K_{RO}l) / /(E_l I_l)$ is utilized. This factor is employed to signify the relative joint stiffness compared to the bending stiffness of the lining segment. Typically, a standard unit length for a lining segment, denoted as $l$, is set at 1 m in calculations.

## Numerical simulations

*Geometric dimensions of the tunnel cross-section*

The sub-rectangular tunnel was first researched by Huang (2018) [2] using full-scale loading tests with outer dimension of 10.7×7.7 m. The tunnel lining is composed of 6 segments (B1, B2, B3, L1, L2 and F block) with a segment thickness of 500 mm. The sub-rectangular tunnel structure is formed by 8 arches, including 2 arches on the sides with an angle of 53 degrees each, and the crown arch and bottom arch with an angle of 45 degrees each, and 4 shoulder arches have an angle of 41 degrees each (Fig. 1).

*Numerical simulation using FEM model*

To investigate the behavior of sub-rectangular (with and without interior columns) the Plaxis 2D program, based on the Finite Element Method (FEM), was employed [27]. The soil parameters were obtained

Table 1

***Geometrical features of tunnel shapes (compiled by authors)***

***Геометрические особенности фигур тоннелей (составлено авторами)***

| Parameters | Sub-rectangular |
|---|---|
| Tunnel Width, $W$ (m) | 10.200 |
| Tunnel Height, $H$ (m) | 7.700 |
| $W/H$ ratio | 1.325 |
| Outer Perimeter | 29.795 |
| Internal Perimeter | 26.654 |
| Internal Area ($m^2$) | 52.926 |
| Outer Area ($m^2$) | 67.038 |

Table 2

***Input parameters (compiled by authors)***

***Входные параметры (составлено авторами)***

| Parameter | Symbol | Value | Unit |
|---|---|---|---|
| **Properties of soil** | | | |
| Density | $\gamma_s$ | 20 | kN/m$^3$ |
| Young's modulus | $E_s$ | 10 | MPa |
| Poisson's ratio | $\nu_s$ | 0.31 | – |
| Internal friction angle | $\varphi$ | 0 | degree |
| Cohesion | $c$ | 0.0256 | MPa |
| The lateral earth pressure coefficient | $K_0$ | $1 - \sin(\varphi)$ | – |
| Overburden | H | 10 | m |
| **Properties of tunnel lining** | | | |
| Material Model | linear elastic | | |
| Young's modulus | $E$ | 35,000 | MPa |
| Density | $\gamma_c$ | 24 | kN/m$^3$ |
| Lining thickness | $t$ | 0.50 | m |

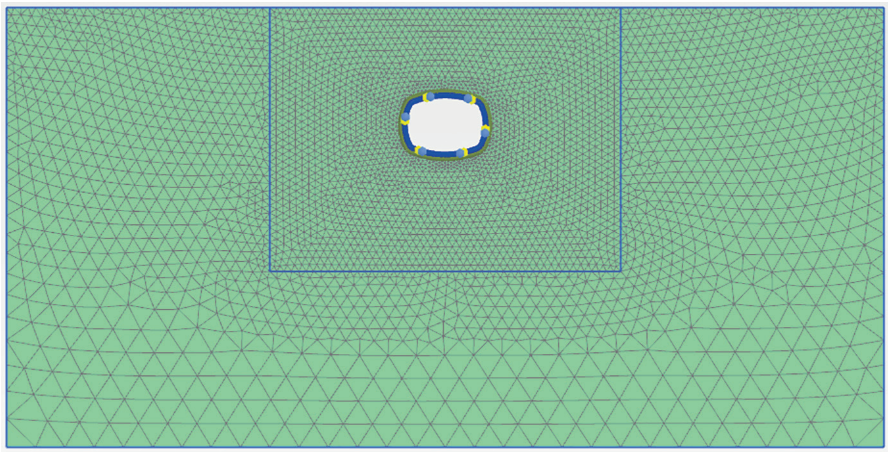

*Fig. 5. Geometry and finite element mesh of the tunnel model (compiled by authors)*
*Рис. 5. Геометрия и конечно-элементная сетка тоннельной модели (составлено авторами)*

from dense clay sand layer of the Hanoi Metro Line 3 tunnel project [17]. The parameters of the tunnels were sourced from the Shanghai Metro Line 4 project [17].

The utilized input parameters are detailed in Tables 1 and Tables 2. The numerical model possesses dimensions of 100 m in width and 50 m in height (comprising approximately 8900 elements and 72 100 nodes. The model boundary conditions are specified as follows: the model bottom is fixed in both vertical and horizontal directions; the horizontal direction of the two sides of the model is fixed; the model top is free.

For simplicity, the soil is represented using the linear elastic-perfect plastic material model based on the Mohr–Coulomb (MC) criteria [28]. Despite the MC model's lower accuracy in depicting soil behavior during settlement compared to the Hardening soil model, as noted by Çelik [29], it remains widely employed in structural behavior studies [30] due to its straightforward input parameters and reduced computational time. The joint between two segments in the segmental lining is simulated using a connected node. This node possesses stiffness equivalent to the rotational stiffness of the joint, as calculated in section 2.3.

## Results and Discussion

### *Influence of the rotational stiffness on the behavior of tunnel lining*

In the reference case, the segmental tunnel lining consists of 6 segments (6 joints) with geometric parameters as depicted in Fig. 1 and detailed in Table 1. The joints are located at the angles of 34°, 84°, 133°, 214°, 264° and 313° measured counterclockwise with respect to the bottom of invert part.

As presented in section 2, to evaluate the influence of the rotational stiffness of the joint $K_{RO}$, a non-dimensional parameter, $\lambda$, is employed. The value of $\lambda$ varies within the range from 0.01 to 1.0. The numerical model results regarding the impact of the rotational stiffness ratio of the joint on internal forces and total displacement are illustrated in Fig. 6.

From Fig. 6, it is evident that when the rotational stiffness ratio is less than 0.3, changes in the internal forces and total displacements are significantly affected by variations in the rotational stiffness. However, when the value of $\lambda$ exceeds 0.3, changes in the rotational stiffness have a smaller impact on both internal forces and total displacements. It is evident that a decrease in the rotational stiffness of the

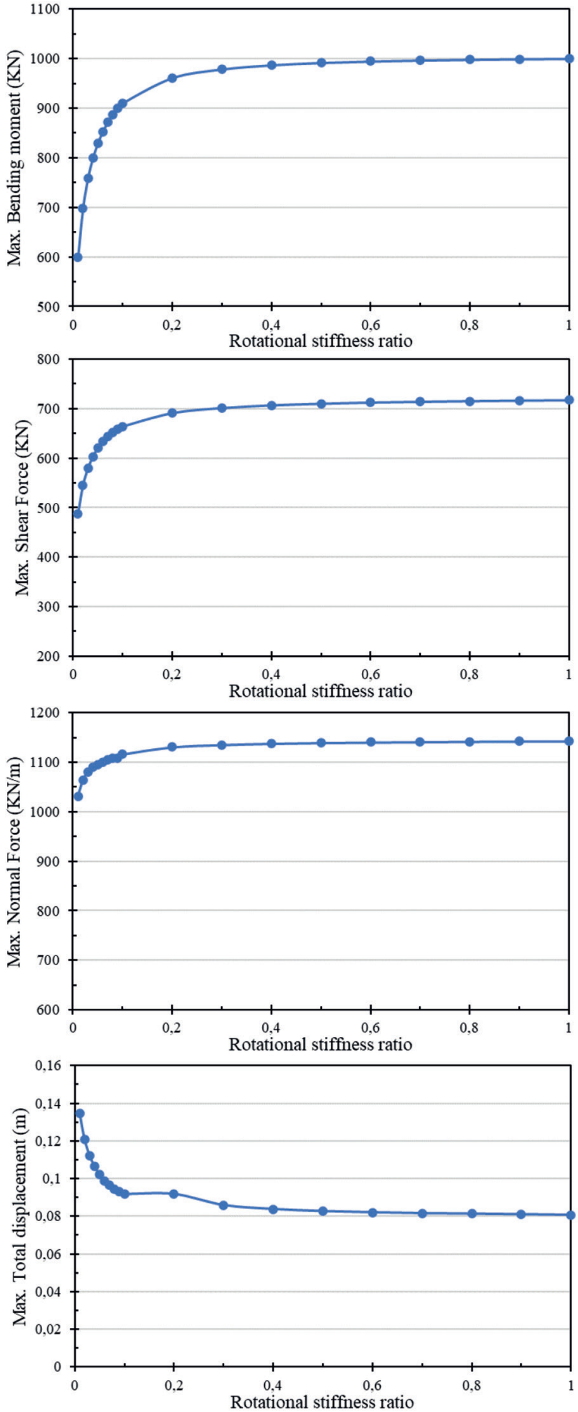


*Fig. 6. Maximum internal force induced in tunnel lining and total displacement (compiled by authors)*

*Рис. 6. Максимальное внутреннее усилие, возникающее в обделке тоннеля, и полное смещение (составлено авторами)*

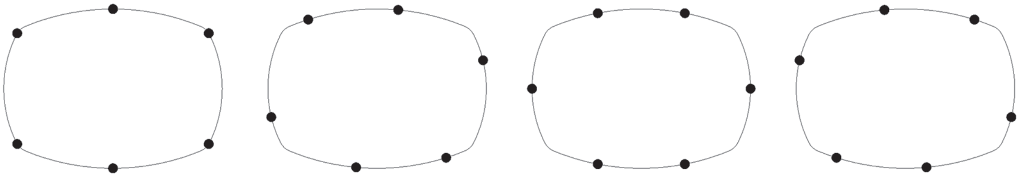

*Fig. 7. Cases of joint location of segmental tunnel lining (compiled by authors)*
*Рис. 7. Случаи стыкового расположения сегментной обделки тоннеля (составлено авторами)*

joint leads to a reduction in the overall stiffness of the tunnel lining, resulting in an increase in total displacement of the tunnel lining. Consequently, the internal forces generated in tunnel lining are reduced. This aligns with previous studies on the influence of the rotational stiffness of the joint on internal forces and displacements in tunnel lining structures with circular cross-sections.

The influence of the rotational stiffness on the maximum bending moment and maximum shear force is greater compared to the maximum axial force.

*Influence of the number and position of joints on the behavior of tunnel lining*

*1. Influence of the position of joints*

In this case, a tunnel lining with 6 joints has been investigated. Assuming each joint is evenly spaced at an angle of 60 ° (angle measured from the center of the tunnel) and the reference joint is located at the crown of the tunnel. The rotational stiffness ratio of the joints is assumed to be 0.1. Soil parameters are determined from Table 2. The joint positions are illustrated in Fig. 7.

Fig. 9 shows the results of bending moments and normal force generated in the segmental lining with 6 joints, considering the joints' orientation angles of 0, 15, 30 and 45° relative to the tunnel crown. While the normal force undergoes minimal change with variations in the angle of the joint relative to the reference joint, the magnitude of the bending moment exhibits much greater variations. In Fig. 9a, the bending moment is the largest when the joint is oriented at 0° relative to the reference joint, while it is the smallest at a 30-degree.

This can be explained as follows: the presence of joints reduces the overall stiffness of the structure. However, the effec-

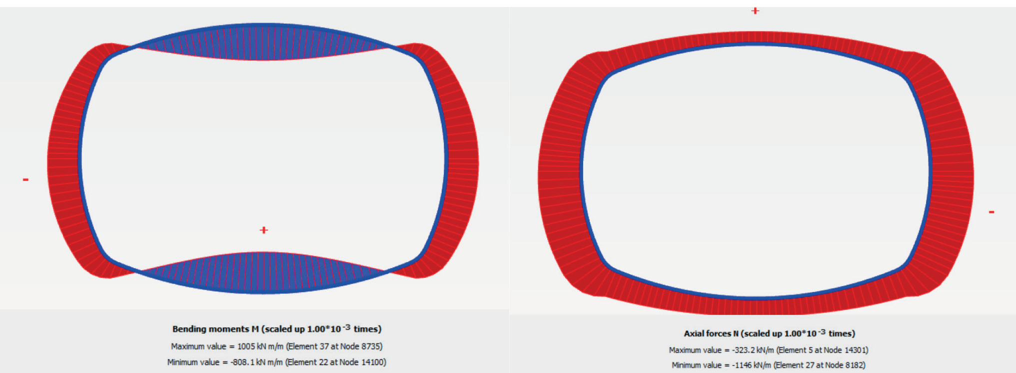


*Fig. 8. Internal force induced in the tunnel lining without joints (compiled by authors)*
*Рис. 8. Внутреннее усилие, возникающее в обделке тоннеля без стыков (составлено авторами)*

tiveness of the joint also depends on its position. In the case of joints at the reference joint position (0°), where the bending moment is the largest (Fig. 8), the introduction of a joint at this position leads to the most significant reduction in the bending moment. However, other joints are located at the shoulders of the tunnel, where the bending moment values are smallest (Fig. 8).

Therefore, the effectiveness of reducing bending moments at these joints is mi-

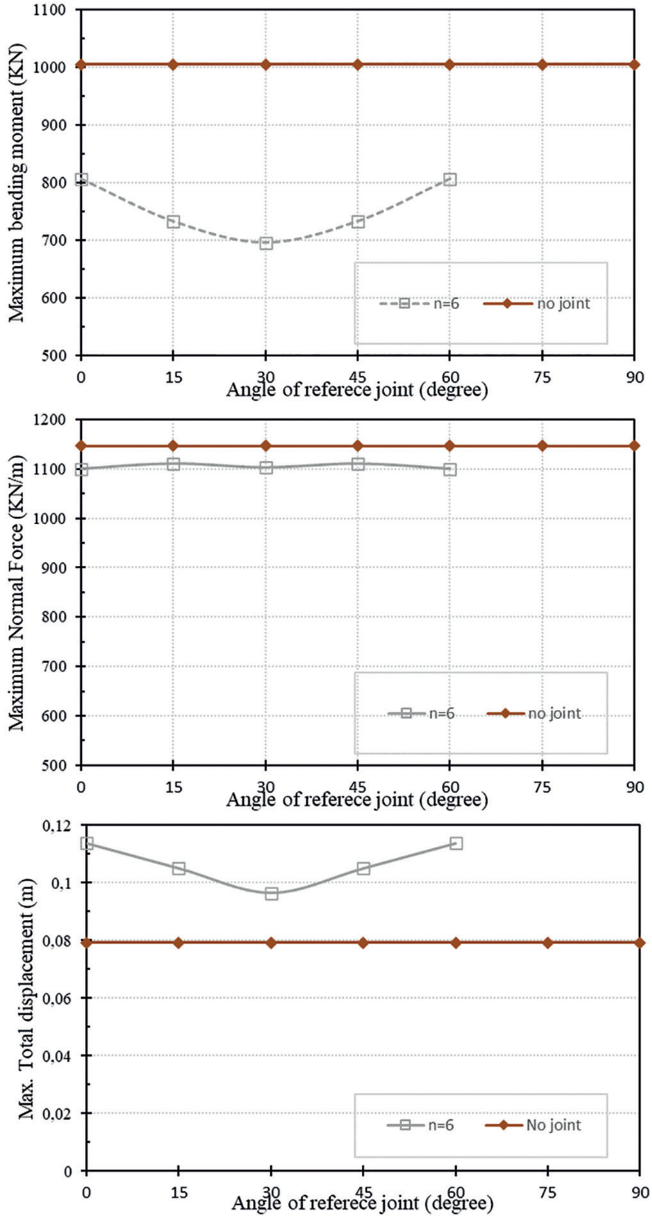


*Fig. 9. Effect of the joint location on the internal force and total displacement (compiled by authors)*
*Рис. 9. Влияние расположения шарнира на внутреннюю силу и полное смещение (составлено авторами)*

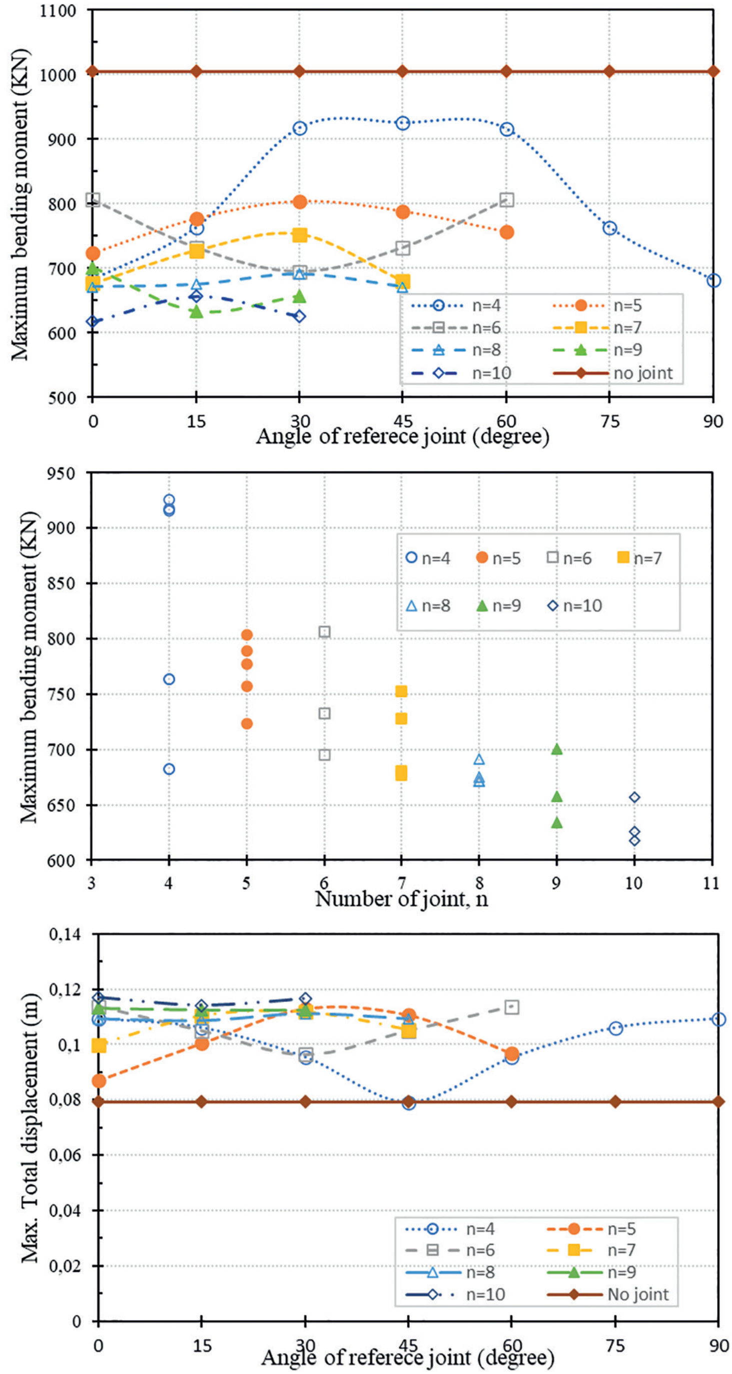


*Fig. 10. Effect of the location and the number of joints on the internal force and total displacement (compiled by authors)*

*Рис. 10. Влияние расположения и количества шарниров на внутреннюю силу и общее смещение (составлено авторами)* nimal. In the case where the angle of the joint and the reference joint is 30°, there are 2 joints at the tunnel crown, 2 joints at the tunnel invert, and one joint on each sidewall.

All joints are positioned at locations with large bending moments. Consequently, the reduction in bending moments within the segmental tunnel lining is most effective.

The total displacement at the joint position relative to the reference angle of 30 degrees is also the smallest, as observed in Fig. 9c.

*Influence of the number of joints*

The number of joints varies in value from 4 to 10, the position of the joints in each case is also changed in steps of 15°.

Fig. 10 shows that an increase in joint number will result in a reduction in the maximum bending moment in the segmental lining and total displacement, its magnitude is also affected by the joint location.

The results indicate that as the number of joints increases, the maximum moment in the tunnel lining decreases. This is evidently attributed to the reduced stiffness of the segmental lining with an increased number of joints, leading to a smaller bending moment generated in the tunnel lining and a larger total displacement.

As per Fig. 10, the impact of the joint orientation is not similar to that of the joint number. Broadly, the discrepancy in the maximum bending moment, stemming from changes in joint orientation, diminishes with an increase in joint number. This implies that with a higher number of joints, the effect of joint orientation diminishes. This occurrence can be elucidated by the shorter span of each segment in a tunnel ring with a greater segment number. Consequently, the loads acting on each segment become nearly uniform in magnitude, irrespective of joint orientation.

The larger the number of joints, the smaller the influence of the joint position on the internal force and total displacement of the tunnel lining. When the number of joints is large, the joints are arranged more evenly and closer on the tunnel boundary. Therefore, when changing the position of the joint, the internal force and total displacement will not change significantly. It can be clearly seen in the case of segmental tunnel linings with 8, 9 and 10 joints in Figs. 10b and 10c.

In the case of a tunnel lining with 4 joints, a shift in joint position results in a substantial alteration in both the maximum bending moment and the maximum total displacement. This underscores the significant impact of joint orientation on the development of the maximum bending moment in a segmental lining. This phenomenon can be attributed to the reduced importance of a joint's influence when it is situated near a point where the bending moment equals zero. Conversely, the influence of a joint on reducing the bending moment becomes more pronounced when the joint is positioned near points of maximum bending moments.

It is intriguing to observe that, for cases with 4 joints, 5 joints, 7 joints, 8 joints and 10 joints, the favorable direction of the joint is characterized by an angle ($\theta$) nearly approaching zero. The reference joint is positioned near the top of the tunnel, and the critical orientation of the joint corresponds to the maximum value of the maximum bending moment at angles $\theta$ of 45°, 30° (5, 7 and 8 joints) and 15°, respectively.

Conversely, in cases where the joint number is 6 joints or 9 joints, the favorable orientation of the joints is indicated by an angle of 30° for the reference joint $\theta$ and 15°, respectively. In these scenarios, the critical orientation of the joints is denoted by an angle of zero degrees for the reference joint (0°).

## Conclusions

In the study, numerous numerical simulations were conducted, exploring variations in the rotational stiffness of the joint, the location of joints, and the number of joints for the segmental lining of the sub-rectangular cross-section.

The following conclusions were drawn:

- The greater the rotational stiffness of the joint, the greater the internal force generated in the tunnel lining and the smaller the total displacement. When the rotational stiffness coefficient is less than 0.3, the change in the stiffness coefficient has a significant influence on the internal force and total displacement;
- The position of the joint significantly affects the internal force in the tunnel lining. When the joint is located in a position with a small bending moment, its influence on the internal force in the tunnel lining is minimal. Conversely, when the joint is located in a position with a large bending moment, its impact on the internal force in the tunnel lining is substantial;
- The number of joints also has a significant impact on the internal force and total displacement of the tunnel lining. The greater the number of joints, the smaller the internal force and the larger the total displacement.

In this study, exclusive focus has been placed on examining the rotational stiffness of the joint, without delving into the effects of normal and shear stiffness. While certain authors have investigated and demonstrated that the normal and shear stiffness of a joint do not significantly affect the internal force induced in circular tunnel linings, it's worth noting that these studies were specific to circular configurations. Moving forward, our research will be expanded to explore the influence of normal and shear stiffness on the behavior of sub-rectangular tunnel linings.

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